Fourier's Law with Exhaustible Reservoirs

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Fourier's Law of Conduction is reviewed, then considered with exhaustible reservoirs. Differential equations are expressed for the rate of flow of thermal energy versus time, Those equations are solved, and formulas are derived for cumulative thermal energy flow versus time. Several cases of reservoirs are presented. Example results are provided.

DISCLAIMER

This paper is exploratory in nature. It is not intended to be a definitive scholarly review of the literature, nor make any claims of new discoveries in physics. It is meant to consider known principles and explore their consequences, new ways to express them, or new applications. These explorations may have uses in and of themselves, or may serve as support for future explorations. Only a limited literature search was prepared while preparing this paper. Apologies are made in advance to any uncredited authors who have already expressed similar ideas.

I. INTRODUCTION

Fourier's Law of Conduction is typically presented in textbooks by a scenario involving a thermal conductor bridging a hot reservoir and cold reservoir, where each reservoir forever maintains a fixed temperature. Yet in real life, this often is not the case. The temperature of one or both reservoirs often changes. Newton's Law of Cooling accounts for the changing temperature of a cooling object. Yet, here, only one object is changing temperature. Also, Newton's Law does not take into account important reservoir characteristics (aside from temperature) or conductor characteristics.

Further, Fourier's Law remains a more frequently taught example, so that extending its applicability will have many educational and practical uses. The author is not proposing any new principle of physics, but rather exploring Fourier's Law from the example of various reservoir characteristics, and expressing the findings in a relatively easy way to utilize.

Heat is defined as the spontaneous flow of energy from one object to another"(p. 17).[1] Since Fourier's Law only concerns thermal conduction, we will only consider the flow of thermal energy. When thermal energy Q flows through a conductor, the rate of that flow is dependent upon both conductor characteristics and the temperature difference ΔT between the thermal reservoirs.[1] See Figure 1. Fourier's Law can be expressed as:

$$\frac{dQ}{dt} = \frac{kA\Delta T}{L} \tag{1}$$

where t is time, k is a constant dependent upon the material comprising the conductor, A is conductor area and L is conductor length.[2]

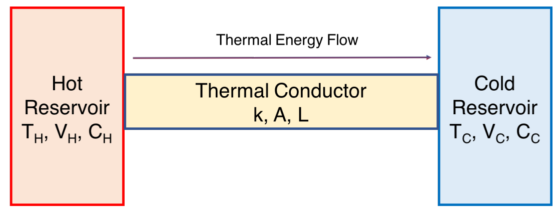

FIG. 1. Thermal conductor bridging thermal reservoirs

This equation assumes that the reservoirs are inexhaustible so that ΔT is a constant.[3] An infinite amount of energy can flow in or out of the reservoirs without changing their temperature. Since the temperature of neither reservoir *ever* changes, regardless of how much thermal energy flows, heat conduction remains a constant over time. Since the reservoir temperatures *never* change, their other characteristics such as volume V and material specific heat capacity C are irrelevant (see later discussion for usage of C). Such reservoirs can be approximated by either an open system, such as a river, or reservoirs that are large and well-circulated. The case where ΔT remains constant, so that the rate of thermal energy flow Q remains constant, can be said to be an example of a system in dynamic equilibrium.

II. EXHAUSTIBLE RESERVOIRS

In many real life cases, reservoirs are substantially exhaustible due to their limited capacity, within time ranges we care about. We shall explore such several cases. We will consider simple, exhaustible reservoirs that are comprised of a single material having a specific heat c. We could express the specific heat capacity (hereinafter referred to as *heat capacity*) in terms of mass, but we will use volume (such as a volume of water) for simplicity and because it is more applicable to simple classroom water bath thermodynamics demonstrations. (Even though an ice water bath involves a phase and hence volume change, the temperature of such ice bath does not change, so both volume and heat capacity are irrelevant for it). Hence, the total heat capacity for each of our hypothetical exhaustible reservoirs is the product of an incompressible volume V of that reservoir times the specific heat capacity C (in units of $energy/(temperature\ x\ volume)$ e.g., J/Km^3) of the material making up the reservoir. For example, for the hot reservoir,

$$T_H(Q_H) = \frac{Q_H}{C_H V_H}, \tag{2}$$

where T_H is a function of Q_H, and H designates the hot reservoir. In all cases we shall consider, C_H and V_H will remain constant. The same relationship and constraints apply to the exhaustible cold reservoir, except that each subscripted value has a C instead of an H to designate cold. The terms hot and cold are by convention. Really, all that is meant is that the hot reservoir is warmer than the cold one. Reservoir temperatures must be expressed in units of absolute temperature such as Kelvin (K). However, in practice, no reservoir is ever at zero K, according to the Third Law of Thermodynamics (p. 94).[1]

A. One Exhaustible Reservoir

An important case is where only one reservoir is exhaustible, but the other one remains inexhaustible. Hence the temperature T of the exhaustible reservoir can change depending upon how much thermal energy flows in or out of it. Volume V and specific heat capacity C must be taken into account for the exhaustible reservoir. This is not an isolated system, although it contains elements of such. The temperature of the exhaustible reservoir can change with time t, so the rate of the flow of thermal energy Q can likewise change. However, once the exhaustible reservoir reaches the temperature of the inexhaustible reservoir, then no more thermal energy will flow, and the system will have reached "static" equilibrium.

Incidentally, the case of one inexhaustible reservoir is analogous to the classic case of a hot object cooling in a cold environment such as air or water, given by Newton's Law of Cooling, where the exhaustible reservoir (typically the hot one) represents the object, and the inexhaustible reservoir (typically the cold one) represents the environment.

We will express Fourier's Law of Conduction as a differential equation with respect to time to get a time-dependent version of Fourier's Law. To solve the equation, we will integrate it with respect to time. We will go through more steps than a traditional paper, so that the current one may, in a sense, serve as a solutions guide. Further, note that units are sometimes implied. For example $\ln|temperature|$ would really mean the natural logarithm of a pure number times the Kelvin unit.

Here, we shall allow the hot reservoir to be exhaustible, so that its temperature T_H can change. The cold reservoir shall remain inexhaustible, so its temperature T_C shall be constant. For this case, V and C shall be for the hot reservoir and are constant. Then, Fourier's Law can be expressed as:

$$\frac{dQ}{dt} = \frac{kA\Big((T_{H_0} - \frac{Q_t}{VC}) - T_C\Big)}{L}. \tag{3}$$

We rearrange the terms,

$$dQ = \frac{kA\big((T_{H_0} - T_C) - \frac{Q_t}{VC}\big)}{L}dt,$$

and combine like terms:

$$\frac{dQ}{(T_{H_0} - T_C) - \frac{Q_t}{VC}} = \frac{kA}{L}dt.$$

Then,

$$\frac{dQ}{(T_{H_0} - T_C) - \frac{Q_t}{VC}} = \frac{kA}{L}dt.$$

Then we integrate both sides:

$$\int \frac{dQ}{(T_{H_0} - T_C) - \frac{Q_t}{VC}} = \frac{kA}{L}\int dt.$$

Upon integration, we obtain:

$$-VC\ln\left|(T_{H_0} - T_C) - \frac{Q_t}{VC}\right| = \frac{kAt}{L} + k_1.$$

Then,

$$\ln\left|(T_{H_0} - T_C) - \frac{Q_t}{VC}\right| = -\frac{kAt}{LVC} - \frac{k_1}{VC}.$$

We raise both sides to e:

$$\left((T_{H_0} - T_C) - \frac{Q_t}{VC}\right) = e^{-\frac{kAt}{LVC} - \frac{k_1}{VC}}.$$

We rearrange to obtain Q_t:

$$\frac{Q_t}{VC} = (T_{H_0} - T_C) - e^{-\frac{kAt}{LVC} - \frac{k_1}{VC}}.$$

$$Q_t = VC(T_{H_0} - T_C) - VCe^{-\frac{kAt}{LVC} - \frac{k_1}{VC}}.$$

We set k_2 to $e^{-\frac{k_1}{VC}}$ to for purposes of simplification:

$$Q_t = VC(T_{H_0} - T_C) - VCk_2e^{-\frac{kAt}{LVC}}. \tag{4}$$

We set the initial values (t = 0) to identify k_2:

$$0 = VC(T_{H_0} - T_C) - VCk_2.$$

We rearrange:

$$k_2 = (T_{H_0} - T_C).$$

We then plug in k_2 into equation 4:

$$Q_t = VC(T_{H_0} - T_C) - VC(T_{H_0} - T_C)e^{-\frac{kAt}{LVC}}.$$

Simplifying, we get our final expression for the equation for the rate of heat flow:

$$Q_t = VC\Big(T_{H_0} - T_C\Big)\Big(1 - e^{-\frac{kAt}{LVC}}\Big). \tag{5}$$

B. Two Exhaustible Reservoirs with Identical Reservoir Dimensions

Next, we shall examine the case where both reservoirs are exhaustible, but where additional reservoir characteristics such as volume V and specific heat capacity C are identical for both reservoirs. The only reason that we assume here that V and C are the same for both reservoirs is that the mathematics is simpler to follow and the results easier to visualize. We will also assume that this is a true closed, isolated system. No thermal energy or matter can enter or leave the overall system. Both temperatures T_H and T_C can change with time as thermal energy enters or exits their reservoir. We assume that V and C remain constant.

Thermal energy will flow from the hot reservoir to the cold reservoir until their temperatures become equal, at which case that system will have achieved "static" equilibrium. Then, Fourier's Law can be expressed as:

$$\frac{dQ}{dt} = \frac{kA\Big(\big(T_{H_0} - \frac{Q_t}{VC}\big) - \big(T_{C_0} + \frac{Q_t}{VC}\big)\Big)}{L}. \qquad (6)$$

We rearrange the terms,

$$\frac{dQ}{dt} = \frac{kA\big(T_{H_0} - \frac{Q_t}{VC} - T_{C_0} - \frac{Q_t}{VC}\big)}{L},$$

and

$$\frac{dQ}{dt} = \frac{kA\big((T_{H_0} - T_{C_0}) - \frac{2Q_t}{VC}\big)}{L}.$$

Then we combine like terms:

$$\frac{dQ}{(T_{H_0} - T_{C_0}) - \frac{2Q_t}{VC}} = \frac{kA}{L}dt.$$

Then we integrate both sides:

$$\int \frac{dQ}{\big((T_{H_0} - T_{C_0}) - \frac{2Q_t}{VC}\big)} = \int \frac{kA}{L}dt.$$

Upon integration, we obtain:

$$-\frac{VC}{2}\ln\left|(T_{H_0} - T_{C_0}) - \frac{2Q_t}{VC}\right| = \frac{kAt}{L} + k_1,$$

and

$$\ln\left|(T_{H_0} - T_{C_0}) - \frac{2Q_t}{VC}\right| = -\frac{2kAt}{LVC} - \frac{2k_1}{VC}.$$

We raise both sides to e:

$$\Big((T_{H_0} - T_{C_0})\Big) - \frac{2Q_t}{VC} = e^{\left(-\frac{2kAt}{LVC} - \frac{2k_1}{VC}\right)}.$$

We rearrange to obtain Q_t:

$$-\frac{2Q_t}{VC} = \Big((T_{H_0} - T_{C_0})\Big) - e^{-\frac{2kAt}{LVC} - \frac{2k_1}{VC}}.$$

$$Q_t = \left(\frac{VC}{2}(T_{H_0} - T_{C_0})\right) - \frac{VC}{2}e^{-\frac{2kAt}{LVC} - \frac{2k_1}{VC}}.$$

We set k_2 to $e^{-\frac{2k_1}{VC}}$ for purposes of simplification:

$$Q_t = \left(\frac{VC}{2}(T_{H_0} - T_{C_0})\right) - \frac{VC}{2}k_2 e^{-\frac{2kAt}{LVC}}. \qquad (7)$$

We set the initial values (t = 0) to identify k_2:

$$0 = \left(\frac{VC}{2}(T_{H_0} - T_{C_0})\right) - k_2.$$

We rearrange:

$$k_2 = \frac{VC}{2}(T_{H_0} - T_{C_0}).$$

We then plug in k_2 into Equation 7:

$$Q_t = \frac{VC}{2}(T_{H_0} - T_{C_0}) - \left(\frac{VC}{2}(T_{H_0} - T_{C_0})\right)e^{-\frac{2kAt}{LVC}}.$$

Simplifying, we get our final expression for the equation for rate of heat flow:

$$Q_t = \frac{VC}{2}\Big(T_{H_0} - T_{C_0}\Big)\Big(1 - e^{-\frac{2kAt}{LVC}}\Big). \qquad (8)$$

C. Two Exhaustible Reservoirs with Differing Reservoir Dimensions

Let us somewhat generalize the previous case. Let us consider the case where both reservoirs are exhaustible, but where additional reservoir characteristics such as volume V and specific heat capacity C are differ for each reservoir (although they still will remain constant over time). Although we again assume that this is a closed, isolated system, so that nether matter nor energy can enter or leave the overall system, this case provides flexibility for additional real life situations. It is slightly more complicated mathematically than the previous case, because there is a volume V and heat capacity C value for each reservoir.

Both temperature T_H and T_C can change. Since V and C can differ for each reservoir, V_H is the volume for the hot reservoir, V_C is the volume for the cold reservoir, C_H is the heat capacity for the hot reservoir, and C_C is the heat capacity for the cold reservoir. Then, Fourier's Law can be then be expressed as:

$$\frac{dQ}{dt} = \frac{kA\Big(\big(T_{H_0} - \frac{Q_t}{V_H C_H}\big) - \big(T_{C_0} + \frac{Q_t}{V_C C_C}\big)\Big)}{L}. \qquad (9)$$

We rearrange the terms,

$$\frac{dQ}{dt} = \frac{kA\Big((T_{H_0} - T_{C_0}) - \big(\frac{Q_t}{V_H C_H} + \frac{Q_t}{V_C C_C}\big)\Big)}{L},$$

and

$$\frac{dQ}{dt} = \frac{kA\Big((T_{H_0} - T_{C_0} - (\frac{V_C C_C + V_H C_H}{V_C C_C V_H C_H})Q_t\Big)}{L}.$$

Then we combine like terms:

$$dQ = \frac{kA\Big((T_{H_0} - T_{C_0}) - (\frac{V_C C_C + V_H C_H}{V_C C_C V_H C_H})Q_t\Big)}{L}dt,$$

and

$$\frac{dQ}{(T_{H_0} - T_{C_0}) - (\frac{V_C C_C + V_H C_H}{V_C C_C V_H C_H})Q_t} = \frac{kA}{L}dt.$$

Then we integrate both sides:

$$\int \frac{dQ}{\Big((T_{H_0} - T_{C_0}) - (\frac{V_C C_C + V_H C_H}{V_C C_C V_H C_H})Q_t\Big)} = \int \frac{kA}{L}dt.$$

Upon integration, we obtain:

$$-\left(\frac{V_C C_C V_H C_H}{V_C C_C + V_H C_H}\right)\ln\left((T_{H_0} - T_{C_0}) - \frac{V_C C_C + V_H C_H}{V_C C_C V_H C_H}Q_t\right) = \frac{kAt}{L} + k_1,$$

and

$$\ln\left((T_{H_0} - T_{C_0}) - \frac{V_C C_C + V_H C_H}{V_C C_C V_H C_H}Q_t\right) = -\frac{kAt}{L}\left(\frac{V_C C_C + V_h C_h}{V_C C_C V_h C_h}\right) - k_1\left(\frac{V_C C_C + V_h C_h}{V_C C_C V_h C_h}\right).$$

We raise both sides to e:

$$\left((T_{H_0} - T_{C_0}) - \frac{V_C C_C + V_H C_H}{V_C C_C V_H C_H}Q_t\right) = e^{-\frac{kAt}{L}\left(\frac{V_C C_C + V_H C_H}{V_C C_C V_H C_H}\right) - k_1\left(\frac{V_C C_C + V_H C_H}{V_C C_C V_H C_H}\right)}.$$

We rearrange to obtain Q_t:

$$\frac{V_C C_C + V_H C_H}{V_C C_C V_H C_H}Q_t = (T_{H_0} - T_{C_0}) - e^{-\frac{kAt}{L}\left(\frac{V_C C_C + V_H C_H}{V_C C_C V_H C_H}\right) - k_1\left(\frac{V_C C_C + V_H C_H}{V_C C_C V_H C_H}\right)}.$$

$$Q_t = \left(\frac{V_C C_C V_H C_H}{V_C C_C + V_H C_H}(T_{H_0} - T_{C_0})\right) - \left(\frac{V_C C_C V_H C_H}{V_C C_C + V_H C_H}\right)e^{-\frac{kAt}{L}\left(\frac{V_C C_C + V_H C_H}{V_C C_C V_H C_H}\right) - k_1\left(\frac{V_C C_C + V_H C_H}{V_C C_C V_H C_H}\right)}.$$

We set k_2 to $e^{-k_1\left(\frac{V_C C_C + V_H C_H}{V_C C_C V_H C_H}\right)}$ for purposes of simplification:

$$Q_t = \left(\frac{V_C C_C V_H C_H}{V_C C_C + V_H C_H}(T_{H_0} - T_{C_0})\right) - \left(\frac{V_C C_C V_H C_H}{V_C C_C + V_H C_H}\right)k_2 e^{-\frac{kAt}{L}\left(\frac{V_C C_C + V_H C_H}{V_C C_C V_H C_H}\right)}. \tag{10}$$

We set the initial values (t = 0) to identify k_2:

$$0 = \left(\frac{V_C C_C V_H C_H}{V_C C_C + V_H C_H}(T_{H_0} - T_{C_0})\right) - \left(\frac{V_C C_C V_H C_H}{V_C C_C + V_H C_H}\right)k_2.$$

We rearrange:

$$k_2 = (T_{H_0} - T_{C_0}).$$

We then plug in k_2 into Equation 10:

$$Q_t = \left(\frac{V_C C_C V_H C_H}{V_C C_C + V_H C_H}(T_{H_0} - T_{C_0})\right) - \left(\frac{V_C C_C V_H C_H}{V_C C_C + V_H C_H}\right)(T_{H_0} - T_{C_0})\Big)e^{-\frac{kAt}{L}\left(\frac{V_C C_C + V_H C_H}{V_C C_C V_H C_H}\right)}.$$

Simplifying, we get our final expression for the equation for rate of heat flow:

$$Q_t = \frac{V_C C_C V_H C_H}{V_C C_C + V_H C_H}\Big(T_{H_0} - T_{C_0}\Big)\left(1 - e^{-\frac{kAt}{L}\left(\frac{V_C C_C + V_H C_H}{V_C C_C V_H C_H}\right)}\right). \tag{11}$$

Despite the quantity of terms in this equation, it is rather simple. The only operative term (independent variable) is the t in the exponent. Everything else is a constant except for the result (dependent variable) Q_t.

III. EXAMPLES AND ANALYSIS

The final equations for all cases considered above have the form:

$$Q(t) = k_r \ \Delta T_0 \ (1 - e^{-\frac{k_c}{k_r} t}),$$

where Q is the cumulative thermal energy flow, k_r combined reservoir characteristics, ΔT_0 the initial temperature difference, and k_c the conductor characteristics. A larger ΔT_0 or k_c will result in a greater Q. The affect of changing k_r is more subtle.

Incidentally, the maximum amount of thermal energy that can *ever* flow is simply:

$$Q(t) = k_r \ \Delta T_0.$$

This quantity represents the thermal potential of the system.

A few examples are provided to illustrate the results. We will cover several cases, where zero, one or both reservoirs are exhaustible. We will also cover cases where the reservoirs have the same and differing characteristics.

In all cases, we will use an initial temperature T_{H_0} of 1000 K for the hot reservoir and an initial temperature T_{C_0} for the cold reservoir. Textbook examples often use 500 K for the hot reservoir and 300 K for the cold one. 300 K is roughly atmospheric temperature. However, 1000 K provides more dramatic results than 500 K and is not unreasonable. It is also faster to mentally compute percentages from 1000 K rather than 500 K. So we will start the hot reservoir at 1000 K.

A. Two Inexhaustible Reservoirs

Here we will examine the case where both reservoirs are inexhaustible. This is the classic case of Fourier's Law. The hot reservoir has a fixed temperature T_H of 1000 K and the cold reservoir has a fixed temperature T_C of 300 K.

Time (s)	T_H (K)	T_C (K)	$\frac{dQ}{dt}$ (W)	Q_t (J)
0	1,000	300	1,400	0
50	1,000	300	1,400	70,000
100	1,000	300	1,400	140,000
150	1,000	300	1,400	210,000
200	1,000	300	1,400	280,000

TABLE I. Two Inexhaustible Reservoirs

The results are shown in Table I. The rate of thermal energy flow dQ/dt does not change over time. However, the cumulative flow of thermal energy flow Q increases with time, and will continue to do so indefinitely. In real life, this case can persist for long periods of time, such as in some layers of atmospheres of Main Sequence stars, but not forever.

B. One Exhaustible Reservoir

Next, let the hot reservoir be exhaustible and the cool reservoir be inexhaustible. This case is not, strictly speaking, an isolated system, but does replicate many industrial processes where heat is being exhausted into a large, well-circulating body of water or the atmosphere. T_C shall be 300K. The hot reservoir will begin at $T_H =$ 1000 K. Table II shows the results for one exhaustible reservoir.

Time (s)	T_H (K)	T_C (K)	$\frac{dQ}{dt}$ (W)	Q_t (J)
0	1,000	300	1393	0
50	558	300	508	44,248
100	395	300	182	60,527
150	335	300	63	66,515
200	313	300	19	68,718

TABLE II. One exhaustible reservoir

The rate of thermal energy flow dQ/dt decreases quickly then slowly with time. This type of change is called exponential decay, and is essentially equivalent to Newton's Law of Cooling. As before, the cumulative flow of thermal energy flow Q increases with time. However, there is a maximum value of Q over the lifetime of this system, which is equal to the difference in initial thermal energies between the reservoirs. Q_t approaches that limit with time.

C. Two Exhaustible Reservoirs with Differing Reservoir Characteristics

Now, we will consider the case where both reservoirs are exhaustible, but with differing reservoir characteristics. Both reservoirs have identical volumes V, but each contains a substance with a different specific heat capacity C. Initially T_C will be 300K, but it can change. The hot reservoir will begin at $T_H = 1000$ K. Table III shows the results for two exhaustible reservoirs with differing reservoir characteristics.

Again, the rate of thermal energy flow dQ/dt decreases quickly then slowly with time. As before, the cumulative flow of thermal energy flow Q increases with time. Again, there is a maximum value of Q over the lifetime of this system, which is equal to an amount that is intermediate to the difference in initial thermal energies between the reservoirs.

Time (s)	T_H (K)	T_C (K)	$\frac{dQ}{dt}$ (W)	Q_t (J)
0	1,000	300	1,400	0
50	587	369	436	41,316
100	458	390	136	54,182
150	418	397	42	58,188
200	406	399	13	59,436

TABLE III. Two exhaustible reservoirs with differing reservoir characteristics

D. Two Exhaustible Reservoirs with Identical Reservoir Characteristics

Finally, we will take a step back and consider the case where both reservoirs are exhaustible, yet with identical reservoir characteristics. Both reservoirs have identical volumes V containing a substance with an identical specific heat capacity C. Initially T_C will be 300K, but it can change. The hot reservoir will begin at $T_H = 1000$ K. The results are shown in Table IV.

Time (s)	T_H (K)	T_C (K)	$\frac{dQ}{dt}$ (W)	Q_t (J)
0	1,000	300	1,345	0
50	697	602	189	30,263
100	656	644	26	34,359
150	651	649	3	34,913
200	650	650	0.5	34,988

TABLE IV. Two exhaustible reservoirs with identical reservoir characteristics

Once again, the rate of thermal energy flow dQ/dt decreases quickly then slowly with time. This is demonstrated in a plot of The Rate of Thermal Energy Flow versus Time shown in Figure 2. In theory, the rate never quite falls to zero, but will continue indefinitely. In reality, the rate will eventually fall to below the level of experimental uncertainty and shall have effectively stopped.

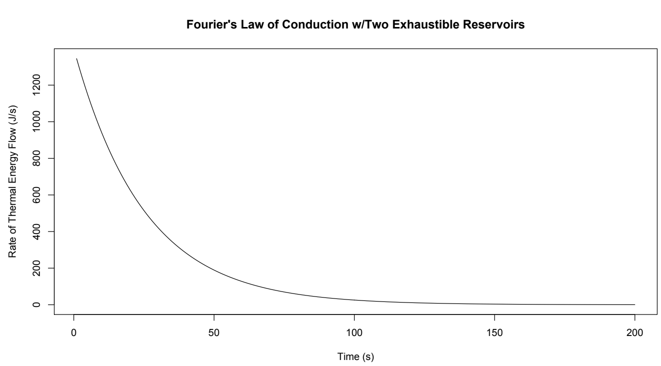

FIG. 2. The Rate of Thermal Energy Flow versus Time

As before, the cumulative flow of thermal energy flow Q increases with time. This is shown in a plot of Cumulative Thermal Energy Flow versus Time is shown in Figure 3. Again, there is a maximum value of Q over the lifetime of this system, which in this case is equal to an amount that is half of the difference of the initial thermal energies between the reservoirs. This total Q is equal to about the amount at which the function eventually approaches $k_r \Delta T_0$.

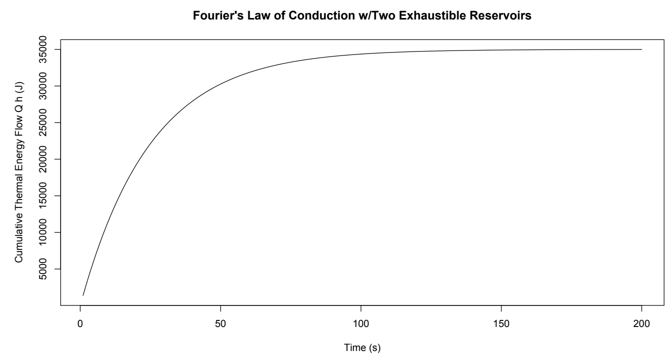

FIG. 3. Cumulative Thermal Energy Flow versus Time

Similar effects within each reservoir can be seen in a plot of the Temperature of Each Reservoir versus Time is shown in Figure 4. Note how the decrease in temperature of the hot reservoir exactly offsets the increase in temperature of the cold reservoir when reservoirs have identical characteristics.

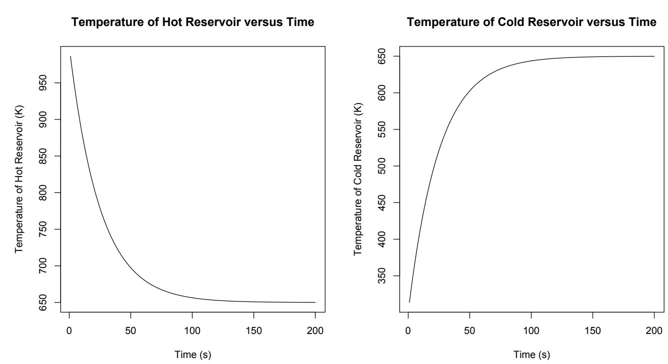

FIG. 4. Temperature of Each Reservoir Thermal Energy Flow versus Time

IV. DISCUSSION

Let us explore some implications. First, it is obvious from Fourier's Law that if the temperature difference increases, then the rate of thermal energy flow shall increase. For all cases, the rate of thermal energy flow doubles if the temperature difference doubles. This is true for all times.

Second, for all cases but that of two inexhaustible reservoirs, there is only a fixed amount of thermal energy that can flow. In the case of two exhaustible reservoirs with identical characteristics, total possible flow of thermal energy is equal to half of the total thermal energy difference between the reservoirs. In the case of two exhaustible reservoirs with differing characteristics, total possible heat flow is some intermediate fraction of the total thermal energy difference between the reservoirs. In the case of one exhaustible reservoir, then the thermal entire energy difference between the reservoirs can flow.

Third, for all cases but that of two inexhaustible reservoirs, the rate of the flow of thermal energy will decrease with time, and inevitably eventually reach a negligible level.

Fourth, there is a distinction between total amount of thermal energy that has flowed, versus the percentage of the total possible thermal energy that has flowed (of the total amount that can ever flow). For example, as noted, increasing the temperature difference will increase the rate of flow, which in turn increases the total that has flowed up to a given point of time. However, an amount of increase in the initial thermal energy of the hot reservoirs required for such a temperature increase means that there is more energy available to flow, which can ultimately require more time to flow, despite the increased rate of flow.

For example, sufficiently increasing the thermal energy in the hot reservoir to double the temperature difference will double the rate of flow. However, the percentage of cumulative thermal energy that has flowed does not double. In fact is can decrease (assuming the product of CV is equal or greater than one). Temperature difference is required as the“force” or tendency to drive flow. Yet the thermal energy required to enable an increased temperature difference is often of greater magnitude than the increased flow created by the temperature difference.

V. CONCLUSION

Fourier's Law, despite being a relatively simple“textbook” example, is either directly applicable to or analogous with a broad range of phenomenon such as electrical conduction through a resistor, water flowing through a pipe, or generally to many situations involving a potential and a constraint. Since the Law is well known and easily explained, it can help provide a large cross-section of the population with an understanding of how these phenomena work.

As mentioned, Fourier's Law provides a superior example than that of Newton's Law of Cooling because both of the reservoirs can be manipulated, such as to either be inexhaustible, exhaustible or to even receive replenishment or continual removal of thermal energy or material. Having explicitly expressing Fourier's Law as a function of time for key time-dependent thermodynamic cases, it is possible to expand this understanding to a much larger class of time-dependent cases in many areas. Hence, Fourier's Law can form the basis for a tremendously broad range of important theoretical and real life situations.

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