# Reproducing Carnot Engines Operating Across Inexhaustible Heat Reservoirs

Mark Ciotola*

*Department of Design and Industry,*

*San Francisco State University, San Francisco, CA 94132*

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## Abstract

A Carnot engine operates across a thermal potential to produce work. This work is utilized to build additional Carnot engines, thus generating growth in consumption and production, similar in form to exponential growth. The impact of engine build cost upon growth is explored. A simulator in the Ruby programming language is discussed.

## I. INTRODUCTION

Heat engines bridge thermal potentials to transform heat flow into work, such as powering a pump, power tool or electric generator. Such work can be utilized to build new heat engines. The paper shall investigate the impact of reproducing heat engines the production of work.

## II. THE CARNOT ENGINE

We shall utilize the Carnot heat engine for our investigation, since its mathematics are simple and unambiguous. The Carnot engine is a hypothetical heat engine that produces the maximum amount of work without producing any net entropy (e.g. Schroeder 2000). It is the most efficient type of heat engine possible under the Second Law of Thermodynamics.

### A. Heat flow and work

Typically, a Carnot Engine is shown to operate between two thermal reservoirs (see Figure 1). The warmer reservoir is designated as "hot" with temperature $T_h$ while the cooler one is designated as "cold" with temperature $T_c$. Heat $Q_h$ flows out of the hot reservoir into the engine, which transforms part of that energy into work $W$, while the engine exhausts the remaining heat $Q_c$ into the cold reservoir. Since the First Law of Thermodynamics states that energy must be conserved, then

$$Q_h = W + Q_c. \tag{1}$$

### B. Efficiency

The proportion of the energy $Q_h$ flowing out of the hot reservoir that is transformed into work $W$ is the efficiency $\epsilon$ of that engine. This is true of any engine. So

$$\epsilon = \frac{W}{Q_h}. \tag{2}$$

However, we can be more specific for a Carnot engine. According to Second Law of Thermodynamics, a quantity called *entropy* cannot decrease for an isolated system. The

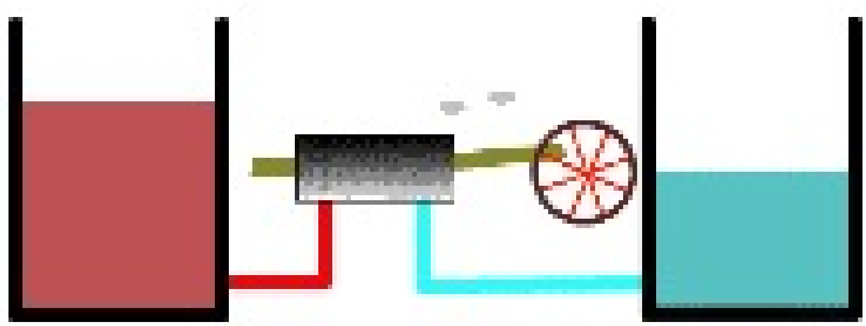

FIG. 1. A Carnot engine bridges a hot reservoir (left) and a cold reservoir(right)

efficiency $\epsilon$ for a Carnot engine is the highest possible efficiency that does not violate that law. Any real life engine could not operate at a higher efficiency, and in many cases will do much worse. Therefore, a Carnot engine neither increases nor decreases total entropy $S$:

$$S_{c_{increase}} - S_{h_{decrease}} = 0. \tag{3}$$

Hence, the foremost characteristic of a Carnot engine is that its efficiency $\epsilon$ is purely a function of the temperatures $T_h$ and $T_c$ of the thermal reservoirs it bridges (for multiple derivations, see Ciotola 2014 #1).

$$\epsilon = 1 - \frac{T_c}{T_h} \tag{4}$$

The proportion of work produced by the engine from heat $Q_h$ removed from the hot reservoir is simply a function of the efficiency $\epsilon$:

$$W = \epsilon Q_h. \tag{5}$$

Now that we know Carnot efficiency, we can determine amount of work a Carnot engine can perform for particular reservoir temperatures:

$$W = \epsilon Q_h = (1 - \frac{T_c}{T_h})Q_h. \tag{6}$$

### C. Entropy

As heat $Q_h$ is removed from the hot reservoir, that reservoir's entropy decreases by $S_h$. Only part of $Q_h$ is transferred to the cold reservoir, but since its temperature is cooler than that of the hot reservoir, the amount of entropy produced per unit of entering heat is higher than that for the hot reservoir. So the entropy decrease experienced by the hot reservoir is exactly offset by the entropy increase in the cold reservoir. In accordance with the Second Law of Thermodynamics, the decrease of entropy in the hot reservoir cannot be higher than the increase in the cold reservoir.

$$S_{h_{decrease}} = -S_{c_{increase}}. \tag{7}$$

Decreasing the thermal energy of a reservoir decreases its entropy. Conversely, increasing its energy increases its entropy. Note that a change of thermal reservoir energy $U$ by an amount of heat $Q$ results in an entropy change $S$ of Q/T, rather than just $Q$.

$$S_{h_{decrease}} = -\frac{Q_h}{T_h}. \tag{8}$$

$$S_{c_{increase}} = \frac{Q_c}{T_c}. \tag{9}$$

## III. USING WORK TO BUILD NEW HEAT ENGINES

The work produced by a Carnot engine can be used to build additional engines. How many engines depends on the work cost to make an engine.

$$New\ engines\ built = \frac{work}{cost\ per\ engine}. \tag{10}$$

As additional engines are built, the total quantity of engines shall increase. As the total quantity increases, and the engines continue to operate, the rate of consumption of heat from the hot reservoir shall increase:

$$Q_h = Engine\ quantity \cdot consumption\ per\ engine. \tag{11}$$

Where both the hot and cold thermal reservoirs are inexhaustible, their temperatures will never change, so that efficiency $\epsilon$ remains constant. Consequently, the rate of work performed will increase proportionately. Since more work is performed, the rate of heat engine construction likewise increase, as the following table shows. One can who this in terms of total heat consumed or in terms of total time passed. We will consider the latter.

In Table I, we begin with one engine in period 1. For convenience, we assume an engine build cost equal to the work produced by an engine in one period (but this can be set arbitrarily, and actual engine costs depend on circumstances). The single engine produces 100 J of work. That is utilized to build an additional engine. So period 2 begins with two engines, that produce 100 J each for a total of 200 J. This is sufficient to build two additional engines, so that period 3 will begin with four engines. As you can see, given these parameters, the quantity of engines, work produced and engines built will double each period. This scenario is shown in Figure 2. Other parameter choices may result in different growth rates.

TABLE I. Engine building and Production

| Period | Engine Quantity | Production | Engines Built |
|---|---|---|---|
| 1 | 1 | 100 J | 1 |
| 2 | 2 | 200 J | 2 |
| 3 | 4 | 400 J | 4 |
| 4 | 8 | 800 J | 8 |

## IV. A MORE DETAILED EXAMPLE

We will next consider a more detailed example. Here, the temperature of the hot reservoir will be 500 K and that of the cold reservoir 300 K. Hence, efficiency will be 40%. We will use a build cost of 1000 J. We will allow fractional engines to produce a proportional amount of work. Figure 3 shows a plot versus time for this scenario. This is a step plot, since new engines (whether whole or fractional) do not start producing until the beginning of the next period.

However, if we reduced the period length to an infinitesimally small amount of time, we

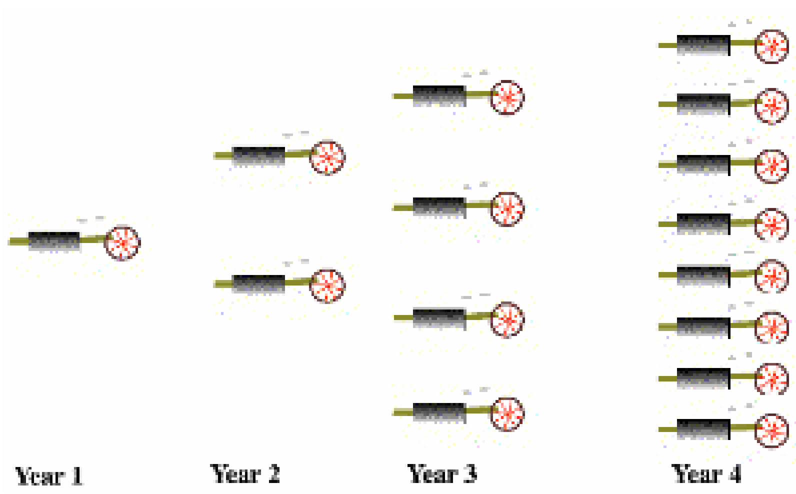


FIG. 2. Reproducing heat engines

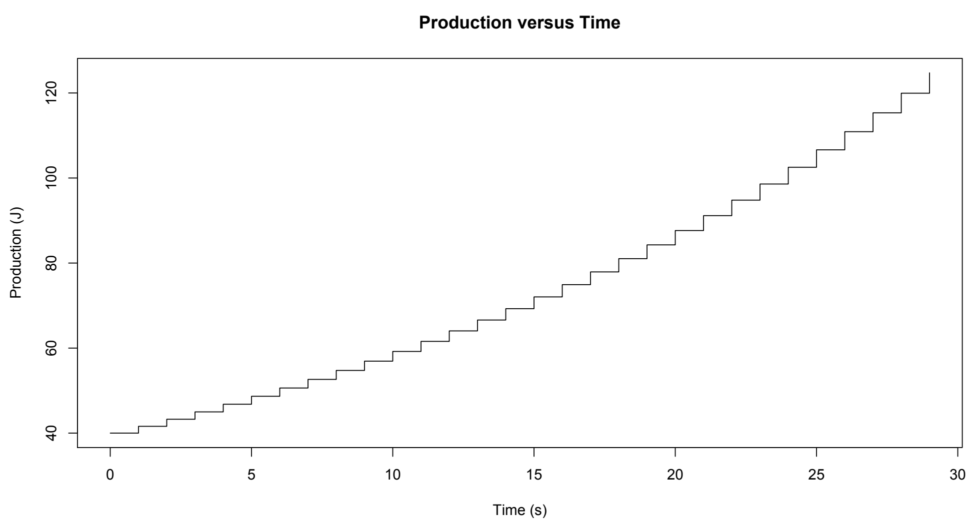


FIG. 3. Engine production versus time

would Note that the plot shows a continuous increase in production, we would get Figure 4. Not the initially gentle increase in production (work) and the steeper rise as time progresses. This plot is similar in form to an exponential growth curve.

## V. IMPACT OF BUILD COST ON GROWTH RATE

We will continue to use the parameters from the last example, except that we will vary engine build costs and compare the results after 30 periods. Figure II shows the high

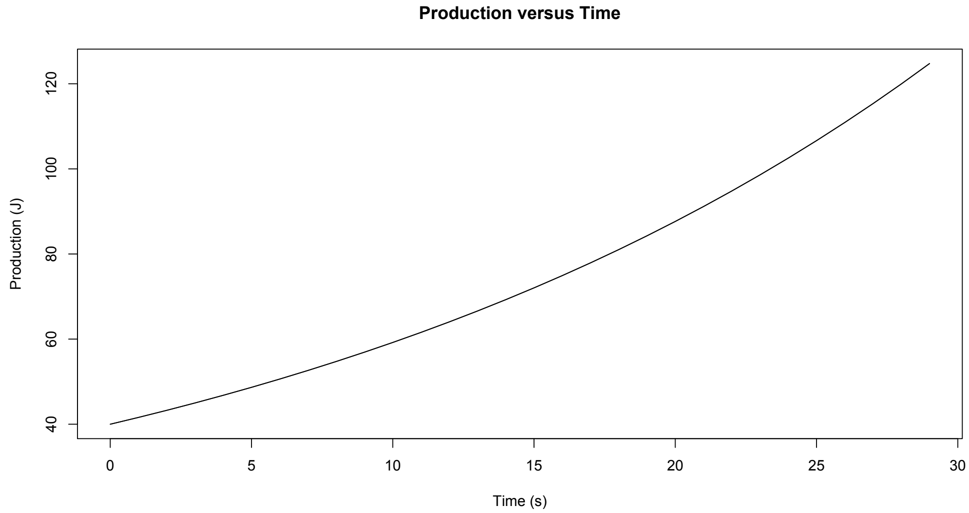


FIG. 4. Engine production versus time

sensitivity of growth of consumption and production upon build cost. As engine build costs decline, the growth of consumption and production approaches infinity.

TABLE II. Results at the end of 30 periods

| Engine Build Cost (J) | Consumption (J) | Engine Quantity | Production (J) |
|---:|---:|---:|---:|
| 100 | 24201.43 | 1,728,673.74 | 691,469.50 |
| 500 | 10.06 | 931.73 | 372.69 |
| 1000 | 3.24 | 311.87 | 124.75 |
| 5000 | 1.27 | 126.00 | 50.40 |

## VI. RUBY COMPUTER LANGUAGE REPRODUCING CARNOT ENGINES SIMULATION

An open-source simulation has been written in the Ruby programming language. Ruby is easy to learn well-suited to introductory simulations. It is also well suited to redeploying such simulations as web applications (Ciotola 2013). View or download the code at: `https://github.com/mciotola/reproducing_carnot_engines_inexhaustible`. Further information may be found at: `http://heatsuite.herokuapp.com/programs/4`.

## VII. CONCLUSIONS

So we can see that reproducing heat engines produces results that are analogous to exponential growth. The case of reproducing heat engines can be generalized to apply to a wide range of phenomena. For example, living organisms such as bacteria operate as engines that consume an energy potential to reproduce. A savings account with compounded interest works in a similar manner.

Limitations of this model is that by assuming inexhaustible reservoirs, we likewise assume a constant efficiency. Neither assumption is true in some cases. We also neglect the lifespan of engines as well as maintenance costs. These could become important factors. Nevertheless, this basic case demonstrates the beginning stages of many real life growth curves.

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* ciotola@sfsu.edu; 1600 Holloway Avenue FA121, San Francisco, CA 94132

[1] Ciotola, Mark, "Exponential Growth Simulator Utilizing the Ruby Language", *HeatSuite Journal* 5 April 2013.

[2] Ciotola, Mark, "A Simulator for the Efficiency and Work of Carnot Engines", *HeatSuite Journal* 12 November 2014.

[3] Daniel V. Schroeder, *An Introduction to Thermal Physics* (2000).